

Question 1:

1)

a) $(p \rightarrow r) \vee (\sim q \rightarrow r)$
 $(\sim p \vee r) \vee (q \vee r)$
 $\sim(p \wedge \sim r) \vee \sim(\sim q \wedge \sim r)$
 $\sim((p \wedge \sim r) \wedge (\sim q \wedge \sim r))$

b) $(p \rightarrow (r \vee q)) \wedge (\sim q \rightarrow (r \vee q))$
 $(\sim p \vee (r \vee q)) \wedge (q \vee (r \vee q))$
 $\sim(p \wedge \sim(r \vee q)) \wedge \sim(\sim q \wedge \sim(r \vee q))$

c) $(\sim(p \rightarrow r) \vee (q \rightarrow r)) \wedge (\sim(q \rightarrow r) \vee (p \rightarrow r))$
 $(\sim(\sim p \vee r) \vee (\sim q \vee r)) \wedge (\sim(\sim q \vee r) \vee (\sim p \vee r))$
 $((p \wedge \sim r) \vee \sim(q \wedge \sim r)) \wedge ((q \wedge \sim r) \vee \sim(p \wedge \sim r))$
 $\sim((p \wedge \sim r) \wedge (q \wedge \sim r)) \wedge (\sim(\sim(q \wedge \sim r) \wedge (p \wedge \sim r)))$

d) $(p \rightarrow (\sim q \vee r)) \leftrightarrow (\sim(p \wedge q) \vee r)$
 $(\sim p \vee (\sim q \vee r)) \leftrightarrow \sim((p \wedge q) \wedge \sim r)$
 $(\sim p \vee \sim(q \wedge \sim r)) \leftrightarrow \sim((p \wedge q) \wedge \sim r)$
 $\sim(p \wedge (q \wedge \sim r)) \leftrightarrow \sim((p \wedge q) \wedge \sim r)$
 $((p \wedge (q \wedge \sim r)) \vee \sim((p \wedge q) \wedge \sim r)) \wedge (((p \wedge q) \wedge \sim r) \vee \sim(p \wedge (q \wedge \sim r)))$
 $\sim((p \wedge (q \wedge \sim r)) \wedge ((p \wedge q) \wedge \sim r)) \wedge \sim(\sim((p \wedge q) \wedge \sim r) \wedge (p \wedge (q \wedge \sim r)))$

2)

Variable	Premise	Conclusion
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a)

p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$p \vee q$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	F

The argument is invalid because when both premises are true the conclusion is false.

b)

p	q	r	p	$(p \rightarrow q)$	$(\sim q \vee r)$	r
T	T	T	T	T	T	T
T	T	F	T	T	F	F
T	F	F	T	F	T	F
F	F	F	F	T	T	F
F	T	T	F	T	T	T
F	F	T	F	T	T	T
F	T	F	F	T	F	F
T	F	T	T	F	T	T

The argument is valid because in the case where all premises are the same value the conclusion is also true.

c)

p	q	r	$(p \vee q)$	$(p \rightarrow \sim q)$	$(p \rightarrow r)$	r
T	T	T	T	F	T	T
T	T	F	T	F	F	F
T	F	F	T	T	F	F
F	F	F	F	T	T	F
F	T	T	T	T	T	T
F	F	T	F	T	T	T
F	T	F	T	T	T	F
T	F	T	T	T	T	T

The argument is invalid because there is a case where all premises are true that the conclusion is also false.

d)

p	q	r	$(p \wedge q) \rightarrow \sim r$	$(p \vee \sim q)$	$\sim q \rightarrow p$	$\sim r$
T	T	T	F	T	T	F
T	T	F	T	T	T	T
T	F	F	T	T	T	T
F	F	F	T	T	F	T
F	T	T	T	F	T	F
F	F	T	T	T	F	F
F	T	F	T	F	T	T
T	F	T	T	T	T	F

The argument is invalid because there is a case where all premises are true that the conclusion is also false.

3.

- a) $\sim p$ Modus Tollens from c and d
 $\sim p \vee q$ by generalization of $\sim p$
r by Modus Ponens on previous statement
 $\sim p \wedge r$ by conjunction of $\sim p$ and r
 $\sim s$ by Modus Ponens of previous statement
therefor $\sim q$ by elimination of statement b
- b) $\sim q$ Modus Tollens from b and d
p Elimination of a from previous statement
 $u \wedge s$ Modus Ponens from $\sim q$ and line e
s Specialization of last line
 $s \rightarrow t$ specialization of c
therefor t by modus ponens of last statement.

4.

- a) A is a knave and B is a knight
- b) C is a knave and D is a knight
- c) There is one knave